THE MATHEMATICS TEACHER

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(Concluded from last issue.)

8. *Fractions.*

(Note: Ability to factor at sight confidently and correctly is of great importance.)

(*a*) *Reduction to Lowest Terms.*—Multiplying or dividing both terms of a fraction by the same number does not change the value of the fraction. Many numerical examples.

Additional connection with arithmetical fractions is established by the numerical check, especially important in reduction of fractions. In checking, values must be substituted which do not give zero denominators.

(*b*) *Reduction to Common Denominator.*—Lowest common multiple explained, with reference to arithmetic. Abundant practice with monomial denominators would precede that with binomial and trinomial denominators, where factoring adds to the difficulties.

(*c*) *Addition and Subtraction.*—Postpone to intermediate algebra such examples as

$$\frac{3}{x-a} - \frac{4a}{a^2-x^2}$$

with the confusing change of signs. Numerical checks may

help to avoid the troublesome error of entirely losing the common denominators.

(*d*) *Multiplication and Division of Fractions.*—This topic may precede (*b*) and (*c*).

Complex fractions should be omitted, except arithmetical ones when needed for checking the solution of an equation like

$$\frac{3x+1}{5x+1}=\frac{5}{7}$$

where $x=\frac{1}{2}$, or for examples in numerical substitution such as finding the values of $(a+b)/(1-ab)$ when $a=\frac{1}{2}$, $b=\frac{1}{3}$.

9. *Fractional Equations and Problems. Literal Equations and Formulas.*

The process of clearing equations of fractions requires very thorough teaching. Equations with decimal coefficients; decimals and percentage; interest. The following formulas should be the basis of part of the work in literal equations, and in substitution:

(1) $br=p$ (percentage),
(2) $b+br=p$ and $b-br=p$ (percentage of increase or decrease),
(3) $i=prt$ or $i=pr\cdot\frac{n}{360}$ (interest),
(4) $i=pr\cdot\frac{n}{365}$ (accurate interest),
(5) $p+prt=a$ (amount).

Solve these formulas, for each of the letters in terms of the others.

(*Note:* T. Percy Nunn avoids the use of x to represent the unknown quantity until after a thorough study of formulas; the solution of simple equations is called by him and by other English authors "changing the subject of a formula.") After solving formulas concerning familiar subject matter such as mensuration for the various letters involved, let the student express each result as a verbal rule, and apply it to specific numerical examples.

Note: Omit special devices of method such as those often recommended for

$$\frac{6x+1}{15}-\frac{2x-4}{7x-16}=\frac{2x-1}{5}$$

and for

$$\frac{1}{x-1}-\frac{1}{x-2}=\frac{1}{x-3}-\frac{1}{x-4}.$$

10. *Ratio and Proportion.*

(*a*) *Ratio.*—Ratio is one of the most valuable topics in elementary mathematics. Emphasize the fact that a ratio, a fraction, and a division are all merely different ways of expressing the same thing, *i.e.*, three ways of indicating division. A decimal fraction or rate per cent. is only a camouflaged ratio. Practice in changing from one form of expression to the other, and in expressing ratio in all possible ways, *e.g.*, "Express as a common fraction and as a decimal to three significant figures the ratio of an inch to a foot, of a foot to a yard, of a yard to a rod, etc."

(*b*) *Partition.*—Dividing a number in a given ratio. Mixtures, alloys, family budgets (*e.g.*, food, rent, clothes, miscellaneous, and savings proportional to 5, 21, 1, 1).

(*c*) *Proportion.*—(1) An equality of two ratios. The product of the means is equal to the product of the extremes (omit "alternation," "inversion," etc.). Numerical and literal equations in the form of proportions.

(2) Scale-drawing. Maps. (Express as a unit ratio the scale: $\frac{1}{4}'' = 1'$, $\frac{3}{4}'' = 1'$, 3 in. = 1 mile, etc. The last answer is called the representative fraction of the map.) Actual house-plans, blue prints, and topographic or city maps should be used in the class-room to ascertain the scale used, measure and estimate distance, etc. The student may prepare a plan of the school or home grounds, an athletic field, a garage, or a floor-plan of a house or of the school room.

(3) Similar figures, protractor; angle measurement and construction. Height of tree by its shadow. Distance to inaccessible point by base-line, angle measurement, and scale drawing. (Examples in reducing and enlarging drawings or pictures, and the pantograph may be used.)

(*d*) *Inverse Ratio.*—The lever. "Work" problems.

11. Graphs.

(*a*) Relative magnitudes from statistics shown by lengths of lines, sizes of rectangles.

(*b*) Circular graph.

Sectors and percentage. A good example would be one showing the make-up of the school in boys and girls of different courses of grades.

(*c*) Graph of a single linear equation in two unknowns.

(*d*) Simultaneous linear equations solved by graphs.

Graphs of inconsistent and dependent equations.

12. *Simultaneous Equations.*

(*a*) Elimination by addition or subtraction.

(*b*) Elimination by substitution.

Besides the usual check some examples may be verified by solving by the other method of elimination or by graphs. Sometimes the same examples may profitably be worked in the three ways.

(*c*) Literal equations.

(*d*) Three unknown quantities.

This is a topic of minor importance on which teachers are prone to linger in the interest of thoroughness. The work with three unknowns would best be sharply limited, say to three lessons.

(*e*) Problems.

Number system and digit puzzles. Sum of the angles of a triangle (the sum to be first derived by measurements with protractor and by paper-folding).

13. *Square Root.*

(*a*) Algebraic expressions.

Only integral expressions which are perfect squares of trinomials. Aim for a genuine understanding of the method, using perhaps $a^2 + 2ab + b^2$ as a type example. Check by multiplication.

(*b*) Numerical square root.

Approximate roots to not over 4 significant figures. Finding the hypotenuse or arm of right angle triangle, $c^2 = a^2 + b^2$.

14. *Exponents.*

(*a*) Illustrate the laws of exponents without formal proof.

(*b*) Meaning of zero and negative exponents:

$$a^2 \div a^2 = - a^{2-2} = a^\circ, \text{ but } a^2/a^2 = 1, \text{ Therefore } a^\circ = 1;$$

$$a^2 \div a^3 = a^{2-3} = a^{-1}, \text{ but } a^2/a^3 = 1/a, \text{ Therefore } a^{-1} = 1/a.$$

(*c*) Meaning of fractional exponents:

$a^{3/2} \times a^{3/2} = a^{3/2 + 3/2} = a^3$, Therefore, because of the definition of square root, $a^{3/2} = \sqrt{a^3}$; also $(a^{1/3})^3 = a^{3/3} = a$, Therefore, $a^{1/3} = \sqrt[3]{a}$.

Examples in exponents should deal with monomials only. They should include practice in writing very large numbers, such as those in astronomy, or very small number by using powers of 10, *e.g.*, three billion miles is 3×10^9; $2 \times 10^{-7} = ?$

15. *Radicals.*

Quadratic surds only, except under (*a*). All work with radicals closely associated with exponents. Fractional exponent and radical sign interchanged when helpful.

(*a*) Removal from the radical of a perfect power. Numerical square root by factoring method.

(*b*) Removal from the radical of a fraction.

(*c*) Four fundamental operations.

The laws understood.

$$2\sqrt{n} + 3\sqrt{n} = 5\sqrt{n},$$

$$\sqrt{m} \times \sqrt{n} = \sqrt{mn},$$

$$\frac{\sqrt{m}}{\sqrt{n}} = \sqrt{m/n},$$

$$(\sqrt{mn})^2 = mn.$$

(*d*) Rationalizing monomial denominators.

Application of (*a*) and (*b*), and (*d*) to sides of right triangles, altitudes of equilateral and isosceles triangles, and diagonals of rectangles. Find approximate value of fractions such $\frac{\sqrt{5}}{\sqrt{3}}$ or $\frac{\sqrt{2} - 1}{\sqrt{2}}$ to four significant figures; do the work with and without rationalization, and make comparisons as to economy of figures and time, and as to degree of accuracy.

16. *Quadratics.*

(*a*) Graphs.

Functions of x such as $y = x^2$, $y = x^3$, etc. Solution of equations of the type of $ax^2 + bx + c = 0$ by drawing the graph of $y = ax^2 + bx + c$.

Graphs help to make clear the nature of the roots, as well as the fact that two roots are possible.

(*b*) Pure quadratics, with problems.

Find the radius of a circle given the area. Include practice in the evaluation of other formulas involving square root such as $c^2 = a^2 + b^2$, $s = \frac{1}{2}gt^2$, $v = \pi$, R^2h, H.P. $= ND^2/2.5$.

(*c*) Solution of general quadratics by completing the square. Not more than two lessons should be taken, with very simple and carefully graded examples sufficient only to make the derivation of the formula intelligible.

(*d*) Solution by formula.

This is the usual method, and one which is always available. Practice in obtaining numerical values of the roots to two places of decimals. Check by substitution, by graphs, or by completing the square. Checking such a root as $2 - \sqrt{5}$ by substitution makes a good example in radicals. Omit difficult literal equations, such as those where either the constant term or one of the coefficients is a binomial.

(*e*) Solution by factoring.

(*f*) Simultaneous equations, one of them linear, the other quadratic. Solution by substitution from the linear equation. Limit work to three lessons. Let each pupil solve four or five examples by graphs also, containing illustrations of the parabola, circle, hyperbola and ellipse: the equations used might be

$$(1)\ \begin{array}{l} y^2 = 4x, \\ x + y = 8; \end{array} \qquad (2)\ \begin{array}{l} x^2 + y^2 = 25, \\ 3x + 4y = 25; \end{array}$$

$$(3)\ \begin{array}{l} xy = 12, \\ y = x + 4; \end{array} \qquad (4)\ \begin{array}{l} x^2 + 4y^2 = 36, \\ x + y + 10 = 0. \end{array}$$

(*g*) Problems involving general quadratics.

New problems may also accompany each method of solution (*c*), (*d*), (*e*), and (*f*).

X. General Notes.

1. *Sequence of Topics.*—The syllabus is not intended to be rigid or restrictive concerning methods, nor concerning such matter as the order of topics. Many teachers, for example, will sometimes teach multiplication before subtraction. Topics will often overlap; preliminary work with a new topic may be carried on while completing the study of the preceding topic. Few recitations should be wholly without practice in verbal and algebraic expression or problems.

2. *Non-essential Topics.*—If necessary further to shorten the course, the following topics of little importance for first-year work could be omitted: algebraic square root, equations with three unknowns, simultaneous quadratics, type *6b* of factoring all fractions with denominators other than monomials.

3. *Definitions and Rules.*—Few definitions or rules need be memorized, but there should be clear oral statements of methods and of explanations; too much should not be expected of beginners.

4. *Written Work.*—There is general agreement that written work of some sort should be handed in by the pupil daily, although the instructor's time will not permit him to grade all the papers carefully.

5. *Mental Algebra.*—There should be frequent practice in mental algebra and mental arithmetic.

6. *Reasonable Answers.*—A thoughtful preliminary guess or rough estimate of the answer and the habit of examining the reasonableness of the actual result will prevent many of the wildly absurd answers so often encountered.

7. *Significant Figures.*—Pupils should develop a certain amount of judgment as to how to carry approximate results. "To the nearest thousandth," "to three significant figures," etc., should be understood from the start. The technique of approximate multiplication and division, etc., probably should not be taught. It belongs in the shop or laboratory, where so much figuring of a certain kind may be required that the student will really appreciate the short methods.

8. *Initiative in Drill Exercises.*—The members of a class may occasionally be encouraged to make up or compose their own

individual examples for drill, *e.g.*, "Write down ten trinomials which are perfect squares, and factor each." "Multiply any quadratic trinomial in x by a binomial, checking by division." "If $x=5$ fill in a second member for each of the following equations and solve:

$3x-2=?$ $5(x+3)-4(x-6)=?$ $(x+2)^2-(x+3)(x-1)=?$

"Make up and solve five different sets of simultaneous equations with $x=4$, $y=3$."

XI. First-Year Mathematics Word List.

Most of the words in the following list should be a part of the pupil's working vocabulary at the end of the first high-school year. Formal definitions need not be aimed at, but rather correct spelling, correct pronunciation and a knowledge of the meaning of the words as shown by their use in simple sentences.

Abbreviation, abscissa, absolute, abstract, accuracy, acre, acute, addend, addition, adjacent, adjoining, affected, aggregate, aggregation, algebra, algebraic, altitude, aliquot, amount, analysis, angle, angular, annulus, annuity, antecedent, applicable, approximate, arc, area, arithmetic, arrangement, ascending, associative, average, axiom, axis.

Balance, base, binomial, bisect, breadth.

Calculate, cancel, capacity, cardinal, census, center, centigrade, centimeter, checking, chord, circumscribe, circumference, classify, coefficient, collect, column, commission, common, commutative, compasses, complement, complete, comples, composite, composition, compound, concave, concentric, concrete, concise, condition, consecutive, consequent, consistent, constant, construction, contents, continuation, convenient, converge, convex, coordinates, correspondence, cross-section, curve, cylinder, cylindrical.

Data, decimal, decrease, deduce, deduction, definition, degree, denominate, denomination, denominator, depth, derive, descending, describe, determine, diagonal, diagram, diameter, difference, digit, dimensions, diminish, discount, dissimilar, distance, distributive, diverge, dividend, divisible, division, divisor.

Elementary, elevation, elimination, ellipse, equal, equation, equiangular, equilateral, equivalent, estimate, evolution, exactly, except, excess, exclude, explanation, exponent, expression, extreme.

Face, factor, Fahrenheit, focus, formula, fraction, function, fundamental.

Gauge, geometric, gram, graphic, gravity, grouping.

Height, hence, hexagon, horizontal, hundredth, hypotenuse.

Identical, identity, imaginary, inclination, incomplete, inconsistent, increase, independent, indeterminate, index, indicate, induction, inequality, innermost, inscribe, inspection, instructor, insurance, integer, integral, intercept, interest, interpretation, interior, inverse, involution, involving, irrational, isosceles, italics.

Kilogram.

Lateral, latitude, length, line, linear, liter, literal, longitude.

Manipulation, mathematics, maturity, maximum, mean, measurement, memorandum, meridian, meter, mill, millimeter, minimum, minuend, minus, miscellaneous, model, monomial, mortgage, multiple, multiplicand, multiplication, multiplier.

Necessary, negative, notation, number, numeration, numerator, numerical.

Oblique, obtuse, occurrence, octagon, omission, omitted, operation, oral, ordinal, ordinate, origin.

Pantograph, parabola, parallel, parenthesis, partial, particularly, partition, pentagon, percentage, perimeter, perpendicular, plotting, point, polygon, polynomial, positive, practical, preceding, precise, preferred, prefix, prime, principal, principle, prism, product, profile, proportion, protractor, pyramid, Pythagoras.

Quadrant, quadratic, quadrilateral, quantity, quart, quarter, quotient.

Radical, radicand, radius, rate, ratio, rational, rationalize, ratios, real, reasonable, reciprocal, rectangle, rectangular, reflex, relative, remainder, representation, respectively, reverse, rod, root.

Satisfy, scale, secondary, sector, segment, separate, series, sextant, significant, similar, simplify, simultaneous, solid, solution, spiral, sphere, square, standard, statement, statistical, straight, subtrahend, subtraction, substitution, succession, sufficient, sum, supplementary, surd, surface, symbol.

Tabulation, tangent, technical, temperature, term, theorem, theory, therefore, thermometer, thousandth, transformation,

translation, transpose, trapezoid, triangle, trinomial, trisect, truncated, typical.

Uniform, unit, unity.

Variable, variation, velocity, verbal, verification, verify, vertex, vertical, vinculum, volume.

Zero, zone.

XII. Bibliography.

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Graphs.

Graphic Method of Presenting Facts. W. C. Brinton. *Engineering News.*
Graphical Methods in Applied Mathematics. G. C. Turner. MacMillan.
Graphic Algebra. A. Schultze. MacMillan.

Practical Mathematics.

Shop Problems in Mathematics. Breckenridge, Mersereau & Moore. Ginn.
Practical Shop Mechanics and Mathematics. J. F. Johnson. Wiley.
Vocational Mathematics. W. Dooley. D. C. Heath.
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Industrial Mathematics. H. W. Marsh. Wiley.
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Industrial Arithmetic for Girls. N. L. Roray. Blakiston's.

Standards.

First Year Algebra Scales. Henry G. Hotz. Teachers College.
Mathematics. Rugg and Clark. University of Chicago.

Note.—For additional bibliography consult the Mathematics Teacher for September, 1915, and volumes of Smith and Young. In the *School Review* for September, 1917, pages 520–526, E. R. Breslich reviews "Recent Literature on Secondary School Mathematics."

There are valuable works by two Englishmen, George St. Lawrence Corson and T. Percy Nunn.

XIII. Note on a College Preparatory Course in Intermediate Algebra.

The following is an outline of the chief additional topics in an intermediate college preparatory course to follow the first-

year course given in this syllabus. Such a skeleton outline may be helpful to many teachers, and it may serve to define our intended scheme for first-year work by showing what is left to be done in order to meet the present college requirements. Such a course needs a full year of work, as is implied in the College Entrance Examination Board recommendation of two units credit for algebra. Much of the time would be used on review topics, with problems of a difficulty more appropriate to the added maturity of the pupils and to the less remote contingency of college entrance.

1. *Factoring.*—(*a*) Difference of two squares, particularly the trinomial type, $a^4 + a^2b^2 + b^4$.

(*b*) Factor theorem.

(*c*) Binomial $x^n + y^n$ studied with the aid of the factor theorem, particularly $x^3 + y^3$.

(*d*) H. C. F. and L. C. M. by factoring, not neglecting factors that differ only in sign, *e.g.*, $a - b$ and $b - a$.

2. *Fraction.*—(*a*) Addition and subtraction including facility with such difficulties of signs as that mentioned in the preceding topic.

(*b*) Complex fractions.

3. *Exponents and Radicals.*—(*a*) Interpretation and manipulation of expressions involving negative, fractional, and zero exponents.

(*b*) Changing surds to the same index.

(*c*) Operations with surds of any order.

(*d*) Rationalization of a binomial surd denominator of the second order.

(*e*) Square root of a binomial quadratic surd.

(*f*) Radical equations.

4. *Quadratic Equations.*—(*a*) Completing the square more thoroughly mastered, including the derivation of the formula.

(*b*) Theory of quadratic equations; the relations between roots and coefficients, and the nature of the roots.

(*c*) Equations in quadratic form.

(*d*) Simultaneous quadratics, including homogeneous equations and those solvable by dividing one equation by the other.

5. *Binomial Theorem* for positive integral exponents including the formula for the *n*th term.

6. *Arithmetical and Geometric Progression.*

Committee: Arthur W. Belcher, Chairman, East Side High School, Newark; James M. Stevens, superintendent of schools, Ocean City; William W. Strader, Dickinson High School, Jersey City; Roy W. Lord, formerly of Plainfield High School; Theodora Skidmore, Barringer High School, Newark.

PROPOSED SYLLABUS IN ALGEBRA.*

By Chas. F. Wheelock,

Assistant Commissioner for Secondary Education.

This syllabus in algebra is in the main a reprint of the tentative syllabus issued in February, 1919. A few changes have been made in response to criticisms received.

This syllabus has not been adopted; it is still in the tentative form and further criticisms are solicited. Such criticisms should be submitted at an early date.

Questions in elementary algebra for the examinations in January and June, 1920, will follow this outline, but they will not include any topics omitted from the syllabus of 1910. Questions in graphs will be introduced but will be optional questions.

Since arithmetical and geometrical progressions are required for admission to many colleges, the examinations in intermediate algebra and in advanced algebra will each contain questions in progressions and logarithms; but these questions will be so arranged that the pupil may exercise his option as to which he shall take in each case.

Elementary Algebra.

In the following syllabus the work of the preceding syllabus is simplified (1) by the omission of several topics of relatively little value; (2) by making the existing requirements more precise; and (3) by improving the classification of the material, particularly with a view to showing the uses of algebra. The work is increased only by the requirement of a subject already taught in practically all schools—the elementary statistical graph, with which everyone must be familiar in order to read intelligently or current literature.

The sequence given in the syllabus has only a general relation to the sequence in which the topics should be presented. For

* The University of the State of New York, the State Department of Education.

example, the equation should be briefly treated early in the course for the purpose of increasing the interest of the pupil and of showing certain of the uses of algebra. In this introduction, it should be applied practically, as in the derivation of one formula from another and in the solution of suitable problems in arithmetic. The subject should be reviewed and extended at a later time.

Algebraic Language.

1. Representation of numeric expressions algebraically.

For example, represent algebraically an even number, an odd number, a fraction, the sum of the squares of two numbers diminished by twice the product of the numbers.

As a second type of illustration, if a boy is x years old at present, how old was he 7 years ago? How old will he be 10 years hence? If his sister is twice as old as the boy is at present, how old will she be 5 years hence?

As a third type, express d dimes as dollars; as cents. If a boy goes to a store with $5 and buys x articles at c cents each, how much change should he receive? Express the results in cents; in dollars.

Since such work is a direct and necessary preparation for the statement of problems in algebraic language, teachers are advised to introduce rapid oral drill of this nature at frequent intervals throughout the course. Simple problems of this nature can easily be made from the problems given in any good oral arithmetic. They should include questions involving price, per cents, rate of speed, and the subjects usually considered in the work in simple equations.

2. Representation of mathematic relations by means of algebraic symbols.

For example, represent algebraically the equality existing between the difference of the squares of two numbers and the product of the sum and the difference of the numbers.

3. Translation of algebraic symbols into words.

For example, translate into words the symbols $a = \pi r^2$, where a stands for the area of a circle and r stands for the radius.

4. Expression of the ordinary rules of arithmetic as algebraic formulas.

For example, express algebraically, using initial letters, the fact that the interest on a sum of money is equal to the product of the annual rate, the number of years, and the principal; thus, $i = prt$.

As a second type, the dividend less the remainder is equal to the product of the divisor and quotient; thus, $d - r = dq$.

5. The correct use of the common terms of algebra.

For example, monomial, polynomial, coefficient, and exponent.

Elementary Graphs.

1. Representation of simple statistics by graphs, only two variables being introduced.

For example, letting the horizontal axis represent the days and the vertical axis the attendance, represent by bars of proper length the class attendance given by the statement that on Monday it was 30; on Tuesday, 32; on Wednesday, 28; on Thursday, 30; on Friday, 26. Also, letting the horizontal axis represent the years and the vertical axis the population, represent by a curve the growth in population of a village given by the statement that in 1900 it was 1200; in 1905 it was 1300; in 1910 it was 1450; and in 1915 it was 1700.

2. Represent graphically the formula $c = 2\pi r$, representing c on the vertical axis and r on the horizontal axis.

3. Ability to interpret a graph.

For example, state what the graph of $c = 2\pi r$ shows with respect to the rate of increase of c as compared with r, and similarly for the graph of $a = \pi r^2$. In the former case it should be observed and stated that c increases at the same rate as r, but in the latter case a increases much more rapidly than r.

For example, from the curve showing growth in population determine the approximate population at any date between the ones given in the table. Also, and conversely, show from the curve the approximate time when the population was represented by a number not given in the table. Also extend the curve properly so as to be able to make approximations concerning the population, say, five years from now.

Negative Numbers.

1. Graphic representation of positive and negative numbers.

For example, represent on a straight line the points corresponding to 0, $+3$, and -4.

2. Concrete illustration of the use of negative numbers.

For example, how are 20 degrees above zero and 30 degrees below zero represented?

3. The ability to perform the four fundamental operations when negative numbers are involved.

For example, find the results of the following: $-7.1+16.3$; $-5\frac{1}{2}+(-9\frac{3}{4})$; $8.3-(2.6)$; $-37-4.3$; $-52-(-16)$; -6.3×2.4; $-3\frac{1}{2}\times(-2\frac{1}{2})$; $-4.8\div 3$; $4.5\div(-3)$; $-6.8\div(-0.2)$.

Fundamental Operations.

1. Addition of monomials and of polynomials.

For example, add $7a^2$, $3a^2$, and $-4a^2$; add $7a^2$, $+3ab-4b^2$ and $16ab+9b^2$; add ax^3, bx^3, and $-3cx^3$.

2. Subtraction involving monomials and polynomials.

For example, from $17x^3-x^2y+2xy^2-y^3$, subtract $-x^3-2x^2y+2xy^2$; from $ax^2+bxy-cy^2$, subtract px^2-qy^2.

3. As special cases of addition and subtraction, the removal of one set or at the most two sets (one within the other) of parentheses or other symbols of aggregation.

For example, remove the following symbols of aggregation and simplify without changing the value of any expression: $a-(2a-b)$; $a^2+(3a-1)$; $3x^2-[2x+(x^2-x)]$; $5.4m^2-[7.2m^2-(m^2-3)+7]$; $a^3+[a^3+(a^2-1)]$.

4. Multiplication of ordinary algebraic expression.

For example, multiply $4a^2b$ by $-0.5ab$; $4a^2+b^2$ by a^2-3b^2; $x^2+2xy+y^2$ by $x^2-2xy+y^2$; x^3+3x^2+3x+1 by x^2+2x+1.

5. As a special case of multiplication the pupil should be able to state at sight simple products of the types indicated by such symbols as $(x+a)(x-a)$, $(a+b)^2$, $(ax+b)^2$, $(ax+b)(ax-b)$, and $(x+a)(x+b)$.

6. Division by a monomial, a binomial or a trinomial.

For example, $a^4b^2c^5\div a^2bc^5$; $(a^2-3a^2b+3ab^2-b^3)\div(a-b)$; $(3x^3+x^2+10x-7)\div(x^2+x-1)$.

7. Checks. Any of the four fundamental operations may be checked by reducing the problem to a corresponding numerical problem.

For example, check the subtraction: $(5x^2-2x-6)-(3x^2+7x-9)=2x^2-9x+3$. Assign to x any value, say 2, the minuend $=10$, the subtrahend $=17$, the difference $=-7$. As $10-17$ does equal -7, the work is probably correct.

8. In all the operations the pupil should be shown the relation of algebra to arithmetic.

For example, the arithmetic addition indicated by the sum of $7t + 3u$ and $5t + 2u$, if $t = 10$ and $u = 1$, is $73 + 52 = 125$; the arithmetic operation indicated by $(3t + u)(4t + 3u) = 12t^2 + 13tu + 3u^2$, if $t = 10$ and $u = 1$, is $31 \times 43 = 1200 + 130 + 3$, or 1333.

9. The evaluation of formulas.

For example, in the formula $a = \frac{1}{2}h(b + b')$, find the value of a when $h = 6$, $b = 4$, and $b' = 5\frac{1}{2}$.

Factoring.

1. Monomial factors.

For example, factor the expression $\frac{1}{2}hbl + \frac{1}{2}hb$.

2. Binomial factors of binomials in the form of $x^2 - y^2$, where x, y, or both x and y may represent either monomials or binomials.

For example, factor $16m^2 - 1$; $25x^2y^2 - 49z^2$; $(2a + b)^2 - c^2$; $(3m^2 + 1)^2 - (3n^2 - 1)^2$.

3. A trinomial square.

For example, factor $4a^2 + 4ab + b^2$, $1 - 6mn + 9m^2n^2$.

4. Trinomials of the form $ax^2 + bx + c$ that may be easily factored.

For example, factor $x^2 - 2x - 63$, $5x^2 + 2x - 7$.

5. Checks. See that your answer is a product, that each factor is prime and that your answer and the original may be put in identical forms by simplifying each.

6. Factoring as an aid in the simplification of numerical work.

For example, arrange $12.7^2 - 9.8^2$ in a form more convenient for numerical computation.

Fractions.

1, Reduction.

For example, reduce the fraction $\frac{16x^2 - 9}{16x^2 + 24x + 9}$ to lowest terms, and the fraction $\frac{17a^2 + 3}{a^2 - 9}$ to a fraction with $a^3 + 3a^2 - 9a - 27$ for its denominator.

2. Permissible changes of sign in a fraction:

For example, $+\frac{2a}{x - 3} = -\frac{2a}{3 - x} = +\frac{-2a}{3 - x}$;

$$+\frac{2}{(x - 1)(x - 2)} = -\frac{-2}{(x - 1)(x - 2)} = \frac{+2}{(1 - x)(2 - x)}$$

$$= -\frac{2}{(1 - x)(x - 2)}, \text{ etc.}$$

3. The four fundamental operations with fractions, the denominators of the given fractions being expressions which may be readily factored.

(1) $$\frac{a+2}{a^2-a}+\frac{2a+1}{a^2-3a+2}-\frac{a-3}{a^2-2a};\quad \frac{m}{a-b}+\frac{n}{b-a}.$$

(2) $$\frac{m^2-1}{m^2+1}\times\frac{m^4+2m^2+1}{m^2+m-2}.$$

(3) $$\frac{P^2-16Q^2}{P^2-25Q^2}\div\frac{P^2+2PQ-8Q^2}{2P^2-11PQ+5Q^2}.$$

Simple (Linear) Equations both numeric and Literal Containing One or Two Unknown Quantities.

1. Solution by any convenient method.

For example, solve the equations $4x-7=3x+4$; $\frac{3}{4}x-5=\frac{7}{8}x+9\frac{1}{2}$; $3.72x-1.74=2.89x+4.26$, carrying the result to the nearest hundredth; $\frac{a}{x}-3b=\frac{c}{3x}+5$; $7x+3y=9$ and $12x-4.8y=13$; $\frac{x}{a}+\frac{y}{b}=c$ and $\frac{3x}{b}-\frac{5y}{a}=7c$.

2. Solution for any letter in the simpler formulas of physics, arithmetic or mensuration, in terms of the other letters.

For example, from the formula $a=p(1+rt)$, find the value of t in terms of a, p, and r.

Since at the present time the use of the formula is found in all lines of business and industry, teachers should recognize that this is one of the most important features in algebra. They are urged to become acquainted with the formulas of a nontechnical nature that are used in local industries and to introduce them into their class work in algebra. Such formulas are often well adapted to the work in graphs.

3. Problems involving equations of the above kind. Such problems are adequately presented in most textbooks, but particular emphasis should be placed upon those that are of a practical nature.

4. Checks.

For example, is 1.7 the root of the equation $2x+7=10.2$?

Roots.

From the beginning pupils should be made familiar with the use of the fractional exponent as well as of the radical sign to indicate roots.

1. Numeric evaluation of radical expressions.

For example, find the value of the expression $a\sqrt{x^2-y^2}$ when $a = 7.2$, $x = 4.8$, and $y = 3$, carrying the result to the nearest tenth; find the value of the expression $\sqrt{a^2bc^3d}$ when $a = 7$, $b = 27$, $c = 25$, and $d = 12$; find the value of $\frac{a}{\sqrt{b}}$ when $a = 5$ and $b = 7$, carrying the result to the nearest hundredth; find the value of $\sqrt{9}+\sqrt{16}-\sqrt[3]{8}-\sqrt[4]{81}+\sqrt{144}$, all such operations being limited to principal roots.

2. Reduction of a radical expression to the simplest form.

For example, reduce $\sqrt{9a^3bc^4d^5}$ to the simplest form for numeric computation.

3. Addition, subtraction and multiplication of simple radical expressions.

For example, add $\sqrt{8a^3b}$ and $\sqrt{18ab^3}$; from $\sqrt{(a+b)(a^2-b^2)}$ take $\sqrt{(a-b)^3}$; multiply $\sqrt{a^2-b^2}$ by $\sqrt{a^2+4ab+3b^2}$. In all cases the results should be expressed in simplest form.

4. Square root of a polynomial, the root having no more than three terms, and the square root of a number.

For example, find the square root of $x^4-2x^3+2x^2-x+\frac{1}{4}$; find the square root of 17.273 to the nearest thousandth.

5. Practical applications to formulas involving radicals.

For example, given the formula $r=\sqrt{\frac{\pi v^3}{2.5}}$, (1) arrange this in a form more convenient for computation; (2) find v in terms of r from the given formula.

Quadratic Equations in One Unknown.

1. Solution of such equations by any convenient method.

For example, solve the equations $x^2+12x-133=0$; $2x^2-15x=27$; $\frac{1}{2}x^2+6\frac{1}{4}x=3\frac{1}{2}$; $.8x^2+.21=1.2x$; $x^4-7x^2+4=0$, carrying irrational results to the nearest tenth.

2. Checks.

For example, find whether 11 and -3.5 are roots of the equation $2x^2-15x=77$.

3. Problems of as practical a nature as possible should be selected, without introducing the technical language of physics or other subjects with which the pupil is not familiar. In general it will be found that this third field is limited.

Simultaneous Equations Involving Quadratics.

1. One simple (linear) equation and one quadratic.

For example, solve the equations $5x - 4y = -1$ and $2x^2 + 3xy - 2y^2 - 22 = 0$; also $x + y = \sqrt{27}$ and $x^2 + xy + y^2 = 21$.

2. Checks.

For example, without solving the equations determine whether $x = 2\sqrt{5}$ and $y = 3\sqrt{5}$ satisfy the equations $x + y = 5\sqrt{5}$ and $3x^2 - 4xy + y^2 = 65$.

See suggestions given under Simultaneous equations, section 3 in the outline for intermediate algebra.

3. Problems in two unknowns as in quadratic equations.

INTERMEDIATE ALGEBRA.

Review.

Pupils will be expected to be familiar with the work in elementary algebra, and the examinations may contain questions in any of the topics given in the syllabus on that subject. For this reason there should be a thorough review of elementary algebra in connection with intermediate algebra.

Elementary Algebra Extended.

1. Factoring will be extended to include the following:

(*a*) The form $a^n - b^n$ when n has any integral value, and $a^n + b^n$ when n is a multiple of any odd number.

For example, factor $x^5 - y^5$, $a^6 - 64$, $a^3 + 8$, $m^6 + n^6$.

(*b*) Simple polynomials of degree not exceeding 4 by the factor theorem.

For example, find the prime factors of the polynomial $m^4 - 4m^3 - m^2 + 16m - 12$.

2. Fractions will be extended to include fractions whose numerators and denominators may contain fractions, such expressions being considered simply as cases of division.

For example, simplify the fraction $\dfrac{\dfrac{x^2 + y^2}{x^2 - y^2}}{\dfrac{x - y}{x + y}}$, the pupil considering this as the simple case of $\dfrac{x^2 + y^2}{x^2 - y^2} \div \dfrac{x - y}{x + y}$, or $\dfrac{x^2 + y^2}{x^2 - y^2} \times \dfrac{x + y}{x - y}$.

3. Roots. This work will be extended to include the rationalization of the denominator of a fraction, this denominator being a monomial root of any order or a binomial quadratic surd.

For example, rationalize the denominators of $\frac{\sqrt{7}}{\sqrt{3}}$, $\frac{ab}{\sqrt[3]{c}}$, $\frac{a+\sqrt{b}}{a-\sqrt{b}}$, and $\frac{7}{\sqrt{3}+2\sqrt{5}}$.

Simultaneous Quadratic Equations.

1. Two homogeneous quadratic equations where one or both are homegeneous except for the absolute term.

For example, solve $6x^2 - 7xy + 2y^2 = 0$ and $4y^2 - 5xy = 6$; also $x^2 + 2xy = 24$ and $11xy - 2y^2 = 60$.

2. Symmetric equations, one of which may be of the third or fourth degree, *readily solvable by quadratics.*

For example, solve the equations $x + y = 5$ and $x^3 + y^3 = 35$.

3. Checks. Be sure the values obtained for the unknown are properly associated in pair. Check each pair by substituting in the original equation (or equations) which was not used immediately to obtain the second value in each pair. In any case there should be as many sets of answers as the product of the degrees of the two equations being solved.

For example, in the equations $x + y = 3$ and $x^3 + y^3 = 117$ which values of x and y should be associated in making up the answer? If the values for x are obtained first, the corresponding values of y will probably be obtained by substituting in equation 1. Hence the correctness of your answer will be tested by substituting in equation 2 only. The same for other equations where there are more than two sets in the answer.

4. Problems involving the above types.

Binomial Theorem.

1. Infer the theorem, without strict proof, for the case of positive integral exponents, finding also the formula for the rth term.

For example, write the expansion of $(a^2 - \frac{1}{2}b)^6$, and also find the 9th term in the expansion of $(1 + r)^{16}$.

2. Apply the theorem to the expansion of simple cases used in financial work.

For example, find the first four terms in the expansion of $(1+r)^{10}$ when $r=0.04\frac{1}{2}$. No problems involving difficulties greater than those in the three cases here stated will be set on examination.

Theory of Quadratic Equations.

1. Assuming that every equation of the type $ax^2+bx+c=0$ has one root, it will have two roots and no more.

2. Relation between the roots and the coefficients.

For example, in the equation $ax^2+bx+c=0$, express the sum of the roots and the product of the roots in terms of a, b, and c. Apply this to determining whether 2 and -7.5 are the roots of the equation $x^2-15=2x$.

3. Formation of an equation from two given roots.

For example, what equation has for its roots $-7\frac{1}{2}$ and 0.4, and no other roots?

4. Nature of the roots.

For examples, find without solving whether the roots of the equation $2.1x^2-3x+1=0$ are rational or irrational, real or imaginary, equal or unequal, positive or negative.

Exponents and Logarithms.

1. Prove the following laws when m and n are positive integers:

$$a^m \times a^n = a^{m+n},$$
$$a^m \div a^n = a^{m-n} \quad m \text{ greater than } n,$$
$$(a^m)^n = a^{mn}.$$

2. Meaning of negative, fractional and zero exponents.

For example, why is it reasonable to take a^{-m} as equivalent to $\frac{1}{a^m}$? $a^{p/r}$ as equivalent to $\sqrt[r]{a^p}$ or $(\sqrt[r]{a})^p$? a^0 as equivalent to 1?

3. The application of this work to the manipulation of surds as already considered.

For example, assuming that the fundamental laws stated in (1) are true for negative, fractional and zero exponents, express such relations as the following by the aid of such exponents:

$$\sqrt[n]{a}\sqrt[n]{b} = \sqrt[n]{ab},$$

$$\sqrt[p]{a} : \sqrt[p]{b} = \sqrt[p]{a:b},$$

$$\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a},$$

$$\sqrt[m]{a})^n = \sqrt[m]{a^n}.$$

4. Logarithms considered as exponents of 10; thus, because $10^2 = 100$, we have $\log 100 = 2$. Explanation of the fundamental laws of logarithms as follows:

$$\log ab = \log a + \log b,$$

$$\log \frac{a}{b} = \log a - \log b,$$

$$\log a^p = p \log a,$$

$$\log \sqrt[r]{a} = \frac{1}{r} \log a.$$

For example, if $a = 10^m$ and $b = 10^n$, then $m = \log a$, $n = \log b$, and $ab = 10^m \cdot 10^n = 10^{m+n}$. Therefore, by definition of logarithm, $\log ab = m + n$, that is, $\log ab = \log a + \log b$.

5. Computation by means of logarithms, using and convenient tables. Interpolation to be included.

For example, find the product of 1.74, 0.38, and 24.6; find $\sqrt[3]{1.73}$; find 1.04^9. The student should be told how many significant figures should be given in the result, depending upon the number of places given in the table that he is using. In general, in any problem that he is likely to be solving, he may expect a close approximation to the fourth significant figure with a four-place table, and to the fifth significant figure with a five-place table.

6. The use of logarithms in the solution of simple cases of expotential equations.

For example, solve the equations $10 = 4^x$; $5.6 = (1.03)^{1+x}$.

Graphs.

1. Graphs of numeric equations of first and second degree.

For example, draw the graphs of the equations $x - 0.2$, $y = 3$ and $x^2 - xy + 3y^2 - 4 = 0$.

2. Application to the solution of numeric equations.

For example, by the use of graphs find to the nearest tenth the roots of the following:

(1) $x + \frac{1}{2}y = 2$ and $x^2 + y^2 = 9$,
(2) $x^2 = 46$ and $x^2 + \frac{1}{4}y = 9$.

3. A more extensive study of the graph in connection with every-day business usage, drill in determining scales, etc.

For example, on the same paper represent graphically the increase in population of the United States and the British Isles from 1800 to 1910, after determining a suitable scale.

ADVANCED ALGEBRA.

Review.

The course in advanced algebra should cover a thorough review of all the topics in elementary algebra and intermediate algebra, with more difficult applications than can be expected in the earlier study of those courses.

For example, the pupil should be prepared on such topics as the following: multiplication and division by detached coefficients, the factoring of such cases as $a^n \pm b^n$ for various values of n, elementary work in continued fractions, all types of work in radicals ordinarily given in elementary textbooks, and the use of logarithms.

Progressions or Series.

1. Deduction of the two fundamental formulas of arithmetic progression, $l = a + (n - 1)d$, and $s = \frac{1}{2}n(a + l)$.

2. From these two formulas derive others by equation methods.

For example, prove that $s = \frac{1}{2}n[2a + (n - 1)d]$.

3. Applications of these fundamental formulas.

For example, find the sum of the first 10 terms of the series $3 + 7 + 11 + 15 \ldots$, and the 15th term of the series $4, -1, -6, -11, \ldots$.

4. Deduction of the two fundamental formulas of geometric progression, $l = ar^{n-1}$ and $s = \frac{a(1 - r^n}{1 - r}$.

5. From these two formulas derive others of a simple nature by equation methods.

For example, show that $s = \frac{a - lr}{1 - r}$.

6. Prove that the limit of s is $\frac{a}{1 - r}$ in an infinite series in which $r < 1$.

7. Application.

For example, show that the limit of the sum of the infinite series 1, ½, ¼, ⅛ . . . is 2.

8. Problems involving the above types.

Permutations and Combinations.

1. Formula for the number of permutations of n dissimilar things taken r at a time.

2. Formula for the number of combinations of n dissimilar things taken r at a time.

3. Applications based on the above formulas.

For example, find the number of permutations of 13 dissimilar things taken 7 at a time, and also the number of combinations of 13 dissimilar things taken 7 at a time. Find the number of ways in which a combination lock of 50 numbers may be set on 4 different numbers.

Complex Numbers.

1. Operations.

For example, multiply $2+3\sqrt{-1}$ by $7-9\sqrt{-1}$, and rationalize the denominator of the fraction $\frac{3+\sqrt{-3}}{3-\sqrt{-3}}$.

2. Graphic representation of a complex number.

For example, represent graphically the three cube roots of 1, that is, the three roots of the equation $x^3-1=0$.

3. Graphic representation of the sum or the difference of two complex numbers.

For example, show graphically that the sum of the three cube roots of 1 is zero.

Theory of Equations.

1. Number of roots.

For example, assuming that ever rational equation has a root, prove that an equation of the nth degree has exactly n roots.

2. Formation of an equation from given roots.

For example, form the equation whose roots are 2, $-\frac{1}{2}$, $3+\sqrt{-1}$, and $3-\sqrt{-1}$.

3. Commensurable roots.

For example, find the roots of the equation $x^5-5x^4-4x^3+20x^2+3x-15=0$.

4. Fractional roots.

For example, find the roots of the equation $12x^3-20x^2-x+6=0$.

5. Imaginary roots.

For example, find the five 5th roots of 1; that is, solve the equation $x^5-1=0$.

6. Checks.

7. Transformation of equations.

(*a*) Of an equation having fractional coefficients into another in which the coefficients are integral, that of the first term being unity.

For example, transform the equation $\frac{3}{4}x^3 - \frac{1}{3}x + 2 = 0$ into an equation of the form $x^3 = ax^2 + bx + c = 0$, a, b and c being integral.

(*b*) Of a complete equation into an equation in which the second term is wanting.

For example, transform the equation $x^4 + 2x^3 + 3x^2 + 4x - 7 = 0$ into an equation of the form $x^4 + ax^2 + bx + c = 0$.

(*c*) Of an equation into an equation in which the roots shall be some multiple of the roots of the given equation, or shall differ from the roots of the given equation by a given number.

For example, transform the equation $x^3 - 7x + 5 = 0$ into an equation whose roots are double the roots of the given equation; or whose roots exceed the roots of the given equation by 3.

8. Descartes's rule of signs.

For example, how many positive roots and how many negative roots may the equation $x^4 - 17x^3 + 65x^2 + 41x - 330 = 0$ have?

Numeric Higher Equations.

1. Horner's method of approximation to the roots of a numerical equation.

For example, find to the nearest ten thousandth the root of the equation $x^4 + 4x^3 - 3x^2 - 28x - 28 = 0$ lying between 2 and 3.

2. Graphs.

For example, by the aid of a graph find to the nearest tenth the roots of the equation $x^4 + 4x^3 - 3x^2 - 28x - 28 = 0$.

3. Checks.

Problems.

Problems should be given showing the use of the formulas and equations studied, and the applications of the subject should be made apparent.

MATHEMATICS FOR THE PHYSIOLOGIST AND PHYSICIAN.*

By H. B. Williams.

It seems to have been very generally assumed that a student intending to enter upon the study of medicine needs little if any preparation in mathematics beyond arithmetic and perhaps a little algebra and geometry. I am informed that students in Columbia College intending to enter the College of Physicians and Surgeons, have been definitely advised by a member of the medical faculty that they would have little if any need for mathematical training and that they would better spend their time in preparation along other lines of study. The physician who gave this advice has been very successful in the practice of medicine and is well known as an educator. I have heard similar views expressed by another well-known and successful New York physician. I presume that the views of these eminent medical men are fairly representative of the opinion of the majority of the medical profession today and inasmuch as the student who contemplates a career in medicine naturally—and very properly—seeks the advice of older men in the profession, it is not surprising that these students come to their professional studies with a mathematical training which is meager in the extreme. This attitude of indifference, sometimes approaching hostility toward the teaching of mathematics to prospective medical students, seems to have sprung in part from the circumstance that many of the shining lights in the medical profession have attained their position in the firmament without having felt the need of more powerful mathematical aids than the average high-school courses afford. We are still hardly beyond the period when similar arguments were supposed to prove that a college education is a superfluous ornament for men who expect to spend their lives in what for want of a more specific term is

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usually called "business." I believe I shall be able to show that although a large measure of usefulness may be attained in the field of medicine by the non-mathematically trained physician, nevertheless the inclusion of such training in his preparation for life will enable him to extend his efforts into a wider field of useful endeavor.

At the outset I would invite your attention to the dual aspect of medicine: it is both a science and an art. One studies the science and the art and practises the latter. The best physicians admittedly are those who while practising their art keep constantly abreast of the advances in the science of medicine and, so far as opportunity and ability permit, contribute by their own efforts to that advancement. To determine what constitutes that necessary and sufficient mathematical preparation which will enable a student to develop into a scientific physician well equipped to cope with the problems of his profession, or into an original investigator in any one of the fundamental medical sciences, we must inquire into the nature of these sciences themselves.

Of these the first to be developed was anatomy, the science of structure, including histology, which deals with structure of such smallness as to require the assistance of the microscope for its study. This department of scientific medicine is almost purely descriptive and since it offers fewer opportunities for the application of mathematics than some of the others, we shall not enter into further discussion of it. Physiology is the science of function; it deals with the manner in which the living structure carries on those activities which distinguish it from nonliving things. No short definition is likely to be complete and satisfactory, but a statement that physiology is the physics and chemistry of living things will direct attention to the phases of the subject which are most pertinent to the present discussion. Pathology is the science of abnormal structure and abnormal function and comprises in the requirements for its successful cultivation all that is included in anatomy and histology as well as all that is included in physiology. Bacteriology, the botany of the most minute plants, many of which have been shown to play an important part in the production of pathological conditions, is in medical schools closely allied with pathology. Bio-

chemistry is a special department of chemistry which concerns itself with those chemical substances and reactions which are associated with the life processes of plants and animals, either as constituents of the living organism, foodstuffs out of which it provides material for growth and repair of waste, or by-products of its life activities. The substances with which the biochemist deals are for the most part of very complex character, and his training in the fundamental branches of his science can scarcely be too thorough.

From this brief statement it will appear without further elaboration that the student of medicine or the investigator in the medical sciences requires training in both physics and chemistry. How much of this training is requisite? If one aspires to do research in either physics or chemistry he will scarcely need argument to convince him that thorough familiarity with the existing state of his chosen science is indispensable. In the cultivation of the medical sciences he will be obliged to investigate phenomena by the same means employed by the physicist and the chemist, but the phenomena will be of a far higher order of complexity. If he is to spend his time profitably, avoiding waste of it and the commission of gross errors, his training in the fundamental science of physics and chemistry, including physical chemistry, should be most thorough. It may be illuminating to point out that the great advances in the medical sciences have not been made by men with scanty training. To mention but one notable example from the field of physical physiology, the work of Helmholtz constitutes probably the largest contribution to this branch of the science ever made by a single individual. Helmholtz was thoroughly trained in the mathematical and physical lore of his day and outside the medical profession he is probably better known for his contributions to physics than for his physiological investigations. It is of more than passing interest to note that most of the work he did in the field of physiology has stood the test of time. To this man, trained originally as a physician and holding chairs in the medical sciences in three famous universities before transferring his activities entirely to the field of physics, we are indebted among other things for our knowledge that the nerve impulse travels at a finite speed, a speed which he measured; for the

theory of quality or timbre of musical sounds; for the invention of the opthalmoscope and opthalmometer and for what to the best of my knowledge remains to this day the most complete treaties on physiological optics. Who shall say that equally fruitful fields do not still await cultivation by scientific physicians who have the forethought and industry to lay in early life the necessary foundation in the fundamental sciences?

The ideal training then for men who are to enter upon careers of investigation in the medical sciences would be such as to make them thoroughly trained theoretical and experimental chemists and physicists with all of the mathematical preparation which this involves, this in addition to the special training in anatomy, zoology, botany and related subjects without which physical and chemical knowledge cannot be applied to the problems of living things. Manifestly the brevity of human life makes this an ideal, the attainment of which is impossible. What then shall we advise? We find that most students who intend to enter eventually upon the study of medicine or upon a professional career in one of the medical sciences elect during their courses in college to study such subjects as general biology, zoology, histology, comparative anatomy, botany, perhaps a little chemistry, usually no more physics and mathematics than is required by the institution they happen to attend for the particular degree to which their course will lead. I have no fault whatever to find with these subjects, all of them are excellent as preparation for a lifeworn to be spent in medicine or in the medical sciences; but are they the subjects to be chosen at the time when students generally elect to take them? I venture to think they are not, at least not for students whose purpose is to prepare for the study of medicine. These students in their professional courses will be given much that would be included in the general biological studies I have mentioned. They will have acquired after the first two years of the medical course such training as would enable them by individual reading to become familiar with any of the subjects mentioned above and should get in a good medical school laboratory training at least as good as they would get in college courses in the biological subjects. In all of their work they will constantly need the fundamental subjects, physics and chemistry, and before they

can make satisfactory progress in the study of either of these it will be necessary for them to secure adequate mathematical preparation. I believe that this foundation should be laid before biological subjects are elected at all.

I realize that there are many physicians who are content to practise the art of medicine without paying much more attention to the progress of the science than is absolutely necessary to enable them to maintain their position among their co-practitioners. They do a necessary work and they have their reward. I am reasonably sure that they would all do their work much better if they were to continue throughout their professional careers to cultivate the science as well as the art. Certainly if a member of my own family were in need of medical attention it would not be to the mere artisans among my colleagues that I should turn for assistance. For the man who is content to practise medicine as a blacksmith shoes horses, I presume all the mathematics necessary would be that simple arithmetic which would enable him to avoid errors in writing prescriptions and in sending out his monthly bills. It is not for these that I make my plea for more extensive preparation in physics, chemistry and such mathematics as is necessary for the profitable study of these. My concern is rather for the student of vision and aspirations who wishes to fit himself for whatever work in his chosen profession opportunity may later throw in his way. To such I would say, secure if possible a full four years of academic training and let it include a large measure of physics, chemistry and physical chemistry, with the mathematical preparation which it is well recognized must accompany these subjects. Then if you find it necessary in later life to branch out into regions of endeavor where you have had no special training, you will be provided with the essential weapons with which any problem in natural science must be attacked, will be able to extend your education without the assistance of a mentor along whatever lines opportunity or inclination indicates.

Perhaps I may be permitted to offer an illustration of the sort of thing which happens all too frequently when physicians with scanty training in the essentials undertake to do original work. A young man working on a problem in chemical physiology had undertaken to make a determination of hippuric acid. After

weighing his product he wished to check its purity by determination of the melting point. Finding the melting point several degrees too high, he made repeated attempts to purify his product, but each time the melting point was the same. It was found that the melting point apparatus he was using involved a considerable stem correction, a matter of which he had never even heard. On applying the correction it appeared that his melting point was very nearly that for pure hippuric acid, and on changing the apparatus for one in which the stem correction was eliminated, his melting point was in agreement with that of other observers for pure hippuric acid with all the precision that could reasonably have been expected. This illustration does not directly involve mathematics, but had this young man received the practical training in physics and chemistry for which I have asked, he would probably have known how to determine melting points before starting original research or at least his training would have been such as to enable him to suspect the cause of discrepancy.

If required to say just what mathematics I consider most important for the scientific physician or the physiologist, I should specify as a minimum, algebra, to include summation of infinite series and the criteria of convergency and divergency, plane and solid geometry, plane trigonometry, the elements of analytical geometry, and at least an introduction to the concepts of the differential and integral calculus. If he has no more than this he will be able to read with at least some hope of understanding many scientific papers which would otherwise remain insoluble riddles. In his work as a practising physician he may never use any part of this training. It is for his use in connection with the underlying sciences during his medical training, in his subsequent scientific reading and in his original research work if he does it, that this preparation will bear fruit. If he can spare time he will make no mistake to include some analytical trigonometry, a thorough course in the calculus and in differential equations, vector analysis, the use of hyperbolic functions, theory of errors, theory of probabilities and the theory of determinants.

It is most unlikely that the average student fitting for the study of medicine will ever be induced to cover such a course

of preparation in mathematics as this, probably for the *average* student the minimum would be all that would be desirable, but the student may at least be advised that he will be well repaid for the time and effort spent in acquiring whatever mathematical training he secures rather than be told that all mathematical study is for him a waste of time. For the student who contemplates a life devoted in the main to investigation either in physiology, pathology or any of the special branches of these large subjects, I would most earnestly advise that the minimum I have mentioned above be by all means secured. At the same time I would urge upon his teachers the advisability of instructing him from the start in the employment of his mathematics as an implement of research. For the student who wishes to use his mathematics principally on problems in physics and physical chemistry, it seems to me it would be much more appropriate for his instructor in mathematics to make use of physical rather than geometrical illustrations wherever possible. Geometry antedates modern physics by many years, geometrical illustrations have become classical and there is no objection to them for students of pure mathematics, but the student of natural science, especially the one inclined toward biological investigations, is apt to need some stimulus in the way of apparent practical application of his mathematical lore in order to tide him over that often uninteresting period where acquisition must precede application.

I believe I have sufficiently emphasized the need for mathematics in connection with the acquisition and use of physics and chemistry. It may be asked whether there are direct applications of mathematics to the problems of the physician and physiologists in his own particular sphere of activity. Strictly speaking I should say not, for careful analysis will show that nearly every problem to which the physician would apply mathematical methods is either a chemical or physical problem in medicine. However, one may mention the possibility of applying the theory of probabilities to medical statistics and of the use of graphic methods to represent biological phenomena. In connection with the latter there is the interesting possibility of fitting equations to the curves expressing these phenomena. I could give abundant examples of the indirect us of mathe-

matics in connection with problems arising in the physiological laboratory. I need only mention the necessity for the physiologist to develop the theory of such instruments as he uses in order to ascertain in what way he may modify them to adapt them the better to a particular investigation. I might give as an instance the development by Einthoven* of the string galvanometer, an instrument widely used both by physicians and physiologists. The essential features of this unique instrument were deduced by Professor Einthoven entirely as a result of mathematical investigations. It may not be known to all of you that there are connected with all life processes, electrical phenomena the study of which has already yielded results of great interest. It is with a view to the successful pursuit of studies along these lines that I have mentioned the hyperbolic trigonometry as desirable. Vector analysis will prove a great convenience in this field of work and probably in others, not to mention the advantages to be derived from a knowledge of it in the study of the fundamental sciences, physics and chemistry. The determination of maxima and minima is an application of the calculus which is frequently used in connection with the re-designing of instruments or in determining how best to work with a particular apparatus. The mere mention of ophthalmology suggests geometrical optics, and one might go on multiplying particular instances.

In conclusion I should like to take this opportunity to bring to the attention of a group of men and women interested in the generality of science, the fact that there is at the present time a lack of adequately trained men in physiology. The younger men have not been entering this field of work in recent years in any considerable numbers. It is a field which offers much in the way of opportunity for original investigation to men of proper training. I believe I have already indicated what I mean by proper training. It has always been true that the best work in physiology has been accomplished by men with a broad foundation in the underlying natural sciences. In the future we shall need more than ever men of the very highest training to carry on physiological investigations. I believe the

* W. Einthoven, "Ein neues Galvanometer, Annalen der Physik," fourth series, Volume 12, 1903.

time is about over when research workers in this field will be recruited in any considerable number from the ranks of the medical profession. I believe also that it is in the best interests of science, general as well as medical, that physiology be regarded as a pure science, to be cultivated as such, not as a mere handmaid of medicine to be studied in medical schools alone. The establishment of physiology as a well-recognized department of graduate university work would undoubtedly help to bring its problems and opportunities to the attention of young men of attainments along the lines of physics and chemistry, and it seems to me that future progress depends largely upon the entrance into this arena of young men trained along the lines I have indicated.

Department of Physiology,
College of Physicians and Surgeons,
New York City.

LOVE MATHEMATICAL.

By Captain Robert C. Gillies.

A was a *constant* of the sterner sex
    Though somewhat *arbitrary*:
He burned with a passion for the sweet Miss *X*
    Who often proved contrary.
She was a *variable*, need we say ?
    And very *independent*;
Though weird was her *locus*, always *A*
    Was her faithful, sure attendant.

Now, *A* had a rival by the name of *Y*,
    Whose actions were distressing:
Miss *X* he seemed to *be governed by*,
    Respect to her confessing.
Often a *radical* but mostly *square*
    He wooed her with great unction:
So that love between *A* and his lady fair
    Was a *discontinuous function*.

A had two friends, young *B* and *C*,
    To whom for consolation
He went in his great anxiety
    To solve this hard *equation*.
"I don't like the sound of $XY + A$,
    $AX - Y$ would be better;
To *eliminate* *Y* I have tried all day,
    Confound the pesky letter."

No way could *B* see, but *C* happened to be
    Of guile a great *exponent*;
"It is not *complex* to annex Miss *X*
    And to vanquish your opponent.
Get her to marry you, if not to-day,
    Next time you have a chance, sir;
Y vanishes, you know, when $X = A$
    So there you have your answer."

B added, "Yes, that is the *rational* way,
    If we must be *explicit*;
Just do as we say and *implicit* obey
    And success? You cannot miss it.

To have a sweet mate is a happy state,
    Celibacy is privative;
When wed you *integrate* and *co-ordinate*
    And joy's the *first derivative*."

A granted that *C*'s was a clever ruse:
    "You're an *acute* observer;
I may be *obtuse*, but there is no use,
    I love her with true fervor;
She's of *normal form* and great *symmetry*,
    But this is my position:
The *witch is irrational*, Q.E.D:
    I've *proved my proposition*."

PRINCETON, N. J.

THEODORE ROOSEVELT.

By HENRY POLK LOWENSTEIN.

Ah! who shall write his history;
    And who shall tell his story?
And who shall name his victory?
    And who shall mark his glory?

He served no master but himself,
    And used the chast'ning rod;
He feared no party, power nor pelf,
    His only Conqueror, God.

Of all great men in this great age,
    In God's most wond'rous plan,
He stands as warrior, seer and sage,
    THE GREAT AMERICAN.

(Dedicated to Quentin Roosevelt, who fell in France bravely fighting for his country.)

NEW BOOKS.

Modern Junior Mathematics. By Marie Gugle. New York: The Gregg Publishing Co. Books I and II. $2.00.

Dr. Charles W. Eliot, in a recent address on the subject "Defects in American Education Revealed by the War," said:

"Arithmetic, algebra and geometry should be taught together from beginning to end, each subject illustrating and illuminating the other two. . . . It should also be the incessant effort of the teacher to relate every lesson to something in the life of the child so that he may see the useful applications of the lesson and how it concerns him."

This text was written with these vital principles always in mind. "Modern Junior Mathematics" is a three-book series planned to give the pupil who does not go to high school and college a working knowledge of mathematics.

This way provides for the teaching, in the first six grades, of the fundamental processes of arithmetic which everyone needs to know in life as well as in school. In grades seven, eight, and nine follows the combination, "junior mathematics," which links on to the elementary arithmetic. It extends the arithmetic to common business practice and simple accounts. It gives the mensuration of common things in one's surroundings. This observational geometry more naturally links on to arithmetic than does algebra. Through the geometry, the need of algebraic symbols arises and they are given a real meaning that was impossible in the old order.

A mathematics course arranged on this basis gives the pupil an insight into the various branches of mathematics, enables them to decide on their future course, and lays the foundation for optional courses in the senior high school. Some of the leading features are:

Instead of being told, the pupil is led by skillful questions to discover facts and relations for himself and to draw his own conclusions.

The elements of bookkeeping are given in Book One, including the use of the cashbook, ledger, sales book, purchase book, trial balance, and profit and loss statement.

The study of mensuration begins with the cube and oblong block with which the pupils are familiar. From these come the square and the rectangle. Diagonals in the latter give triangles, which lead to a study of angles and lines. This is the reverse of that order given in other texts on observational geometry, which usually follow the traditional order of formal, abstract geometry, beginning with lines and angles, and ending with the mensuration of the rectangle.

Algebraic symbols are introduced naturally through the formulas of mensuration and through rectangular drawings with unmeasured lines.

The seventh grade pupil wants something new. Book One opens with the graph, which is not only new but intensely interesting. Graphs are not given in an isolated chapter to be taught or not as the teacher wishes, but they are used throughout to illuminate the different phases of the subject.

The problems are real; therefore they appeal strongly to children.

Actual business practices are taught, as reading interest from tables, etc.

The abundance of comparison and design appeals to children for they are primarily "doers" at this age.

The habit of thrift is developed by planning budgets and keeping accounts.

Speed and accuracy in computation with small numbers are emphasized.

Useful short cuts are given.

Definitions are introduced only as needed.

The series is adapted for use in either the 8-4 or the 6-3-3 plan of organization.

General Mathematics. By RALEIGH SCHORLING and WILLIAM DAVID REEVE. Boston: Ginn and Company. Pp. xv + 488. $1.48.

The authors of this text have made a consistent effort to organize a first course in mathematics, such that it will meet the general needs of one who goes no further with the subject, and at the same time will serve as a good foundation for more advanced study.

The book includes algebra through the quadratic equation; geometry, particularly construction and measurement; trigonometric measurement by right triangles; logarithms, and use of the slide rule.

It omits much of the unnecessary complications of algebra, and presents the entire subject in an interesting, as well as accurate, way.

Examination Exercises in Algebra. By IRVING O. SCOTT. Boston: Allyn and Bacon. Pp. xii + 276. $1.60.

This excellent collection of examination questions has been compiled from nearly five hundred examination papers, sent by over seventy colleges located all over the United States.

The questions are well arranged and indexed, and should prove a great help to teachers,—although they, of course, contain a large share of the complications that are gradually being eliminated.

The Book of the Damned. By CHARLES FORT. New York: Boni and Liveright. Pp. 298. $1.90 net.

By damned the author here means excluded, and the fundamental theory of the book is that modern science is exclusionist and intolerantly orthodox, so that it refuses to admit the reality of many important scientific phenomena, or failing that, it contents itself with thoroughly inadequate explanations which evade the issue and miss the point.

The author spent twelve years of patient research in which time he amassed a body of evidence which, he contends, has hitherto been ignored by scientists. Whether the reader will agree with the arguments or not the volume will furnish stimulating reading.

Verse for Patriots. Compiled by JEAN BROADHURST and CLARA L. RHODES. Philadelphia: J. B. Lippincott Co. Pp. 367.

Much is being done these days in the way of teaching Americanism and one of the good ways of doing this is to stir the emotions and fire the imagination by proper ideals in poetic form. The compilers of this volume have made a fine collection for this purpose.

The New American Thrift. Philadelphia: The Annals of the American Academy of Political and Social Science, Vol. 87, No. 176.

It is to be hoped that one result of the war will be to dissociate from the world thrift the unfortunate connotations which it has carried to the minds of many Americans, and have it stand out in its true significance. To this end we have thrift campaigns and movements to get the people to understand its true meaning and real practice. This issue of the Annals is given up to a very fine and comprehensive series of articles by prominent authorities and should do much to educate its readers in this direction. Read it by all means.

NOTES AND NEWS.

THE morning session of the holiday meeting of the Syracuse Section was given to the discussion of the qualifications of the teachers of high-school mathematics. The discussion was opened by A. N. Smith, of Colgate University, who spoke substantially as follows:

As a teacher of college mathematics, I find my knowledge of the difficulties under which the teachers of secondary mathematics labor is largely a matter of conjecture based on the work done by the average freshman and such information, more or less dependable, as I am able to obtain directly from my pupils. I have endeavored, however, in preparation for this meeting to obtain the opinions of representative college and high-school teachers as to the actual and the desirable preparation of those who teach the mathematics of our high schools.

In carrying out this plan, I have asked the following questions of the college instructors:

"1. What defects do you notice in the preparatory training of freshmen?

"2. From your experience, what do you consider to be the principal defects in the high-school teaching?

"3. What preparation do you think should be required of those who teach mathematics through trigonometry?

"4. Do you favor the requiring of a certificate of proficiency?"

Replies to these questions were received from Penn State, Dartmouth, Cornell, Vassar, Hobart, Hamilton, Wells, Hillsdale, The University of Chicago, and Wisconsin. The prevailing opinion on the first question is well expressed in this reply from one institution:

"We have little to say in praise of the preparatory training of the freshmen. The average freshman knows very little about mathematics and what little he does know, he does not know well. His arithmetic and his algebra are usually atrocious.

Whatever he can do, he does in a purely formal manner with no insight into the reasons for the processes employed."

As to the underlying causes for these defects, I quote again from the same letter:

"To little attention is paid to the maintaining of a proper view point and to an understanding of the fundamental principles underlying the processes employed. Too often the reason for the poor teaching is that the teacher can not impart the proper view point and understanding because he does not have it himself."

It was to be expected that the college men would be unanimous in considering a college course with mathematics as a major as a minimum preparation for high-school teaching. Opinions as to the advisability of a certificate were about evenly divided in as much as some doubted the efficacy of such a requirement.

Questionaires were also sent to some three hundred high-school teachers of New York State to which one hundred forty replies were received. The conclusions reached from a study of these replies were as follows:

69 per cent. were college graduates.

21 per cent. were normal graduates.

60 per cent. were specializing in mathematics.

40 per cent. were teaching mathematics "on the side."

66 per cent. of the college graduates were specialists in mathematics.

77 per cent. of the specialists were college graduates.

The training received by these teachers in mathematical courses was as follows:

21 per cent. had taken normal training courses in mathematics.

13 per cent. had taken one year of college mathematics.

25 per cent. had taken two years of college mathematics.

11 per cent. had taken college courses in pedagogy.

16 per cent. made no report.

7 per cent. had no preparation of any sort in mathematics.

10 per cent. considered that the only necessary preparation was to have covered the ground in advance of teaching.

10 per cent. would require a course in methods in addition.

10 per cent. would require one year of college mathematics.

27 per cent. would require mathematics through the calculus.
8 per cent. would require a college course, no mention being made of special courses in mathematics.

Following the presentation of these facts the meeting was opened for general discussion as to the advisability of a qualifying certificate for the teaching of high-school mathematics in New York State. Among those present who spoke was Mr. Seymour of the State Education Department. The difficulties and objections raised to such a plan were well expressed by an extract from a letter in which the writer said:

"Personally I am very much in favor of establishing some sort of qualification test in all high-school subjects. However, just at present there are certain practical difficulties. There is a scarcity of teachers, the salaries are unattractive, etc. These questions vitally affect the small schools. In fact, until the State finds some way of reorganizing the small high schools where one teacher often has to take care of six or more subjects, there is little hope.

"I have often made a plea for separate grades of certificates; *e. g.*, a rural school may be left free to accept a certain minimum certificate (proportional to the small salary paid) but a large town or city should insist on better qualifications and demand a major certificate. Thus in a small school a teacher might hold a license for two or more minors but in a large school a major should be demanded."

The consensus of opinion of those present as a result of the discussion is indicated in the following motions:

1. It was voted to recommend to the State Department of Education the granting of a qualifying certificate to those desiring to teach mathematics in the high schools of the State, the holding of a certificate not being compulsory at present.

2. It was voted to appoint a committee to formulate what should be considered a minimum for the granting of such a certificate.

Meeting of Association of Teachers of Secondary Mathematics.

On January 30 and 31, 1920, the annual meeting of the Association of Teachers of Secondary Mathematics in North Caro-

lina was held at the North Carolina College for Women, Greensboro, N. C.

The women who were in attendance were entertained while in Greensboro by the college. More than forty teachers were present. On the afternoon of January 30, the college gave an informal tea in honor of the Association.

Mr. W. W. Rankin, professor of mathematics, University of North Carolina, who is the executive secretary of the Association, is on a year's leave of absence for graduate work at Columbia University, N. Y. His absence necessitated double work for the president, Miss Cora Strong, of the North Carolina College for Women, in planning the program and in making other arrangements for the meeting.

Professor L. C. Karpinski, of the University of Michigan, was the distinguished speaker invited to address the Association. On Friday evening he made his first address, an illustrated lecture on "The History of Algebra." This meeting was open to all the college students of the city and also to the students of the nearby colleges. His second lecture, "The Methods and Aims in the Study of Mathematics," was given on Saturday morning. On Saturday afternoon, he spoke on "The Practical Applications of High-School Mathematics." He made the teachers realize how intensely alive and useful their science is, showing how the plan of a large auditorium, the reflector of an automobile, the arch of Hell Gate Bridge, the path of a projectile and the orbit of a comet are all reflections of an algebraic equation, and how the price of a railroad ticket may really be said to depend upon the binominal theorem.

Miss Irene Templeton gave a summary of the recent preliminary report of the *National Committee on Mathematical Requirements* on the "Reorganization of the First Courses in Secondary School Mathematics." A committee was appointed to get the consensus of opinion of the teachers of secondary mathematics in North Carolina and communicate with Professor J. W. Young, chairman of the national committee.

The following officers were elected for the year 1920: *President,* Mr. A. W. Hobbs, University of North Carolina; *First Vice-president,* Mr. T. C. Amick, Elon College, Elon, N. C.;

Second Vice-president, Miss Fannie B. Robertson, Fayetteville High School, Fayetteville, N. C.; *Recording Secretary,* Miss Birdie McKinney, Teachers Training School, Greenville, N. C.; *Permanent Secretary,* Mr. W. W. Rankin, University of North Carolina.

MARIA A. GRAHAM,
Recording Secretary, 1919.

At the last meeting of the General Education Board in New York on February 28, the sum of $25,000 was appropriated for the use of the National Committee on Mathematical Requirements to continue its work for the year beginning July 1, 1920.

A preliminary report on "The Reorganization of the First Courses in Secondary School Mathematics" was published for the Committee by the U. S. Bureau of Education about the middle of February. It has been distributed widely. Copies of the report have gone to all the state departments of education, to all county and district superintendents in the United States and to all city superintendents in cities and towns of over 2,500 population. It has been sent to all the normal schools in the country, to some 1,500 libraries and to almost 300 periodicals and newspapers. In addition it has been sent to about 4,500 individuals, the names and addresses of which were furnished the Bureau of Education by the National Committee. This list of individuals consists chiefly of teachers of mathematics and principals of schools throughout the country. Additions to this mailing list to secure future copies of the reports of the Committee can still be made. Individuals interested in securing these reports should send their names and addresses to the Chairman of the Committee (J. W. Young, Hanover, N. H.).

A subcommittee consisting of Professor C. N. Moore of the University of Cincinnati, Mr. W. F. Downey of Boston and Miss Eula Weeks of St. Louis has been appointed to prepare a report for the Committee on Elective Courses in Mathematics for Secondary Schools. Any material or suggestions for this report may be sent directly to the chairman of the subcommittee.

The recent work of the National Committte had a place on the program of the organization meeting of the National Council of Teachers of Mathematics held in Cleveland on February 24 in

connection with the meeting of the Department of Superintendence of the National Education Association. The meeting for the organization of the National Council was enthusiastically attended. A constitution was adopted and officers and an executive committee elected. Mr. J. A. Foberg of the National Committee on Mathematical Requirements was elected Secretary-Treasurer of the National Council.

Recent meetings of teachers at which the reports of the National Committee have been discussed have taken place in New York City, Cincinnati, San Francisco, Cleveland, Oklahoma, Philadelphia, Springfield (Mass.), Providence (R. I.), Meetings in April will take place in Alabama, Illinois, Iowa, Michigan and Kentucky.

Congrès International des Mathématiciens

Dans sa première réunion du 24 Décembre 1919, le Comilé National Français des Mathématiques, après avoir choisi comme Président d'Honneur: M. Jordan; Président: M. Picard; Vice-Présidents: MM. Appell, Borel, Lecornu, Le Roux; Secrétaire général: M. Kœnigs; Secrétaire: M. Galbrun; Trésorier: M. Maluski, s'est occupé de l'organisation du Congrès des Mathématiciens qui, suivant le vœu émis à Bruxelles par l'Union Internationale provisoire des Mathématiciens[1] doit se tenir à Strasbourg, en 1920.

Il a l'honneur d'inviter à participer aux travaux du Congrès les Mathématiciens des Nations de l'Entente et ceux des Nations neutres dont la liste a été arrêtée par la Troisième Conférence interralliée des Académies, tenue à Bruxelles en Juillet 1919. Il leur fait savoir qu'il sera reconnaissant à chacun de lui adresser le plus tôt possible son adhésion personnelle pour des raisons d'organisation faciles à comprendre.

La date de l'ouverture du Congrès sera fixée au 22 Septembre.

[1] L'Union Internationale provisoire, fondée par les Mathématiciens présents à la Conférence interralliée des Académies scientifiques, tenue à Bruxelles en Juillet 1919, a constituté son bureau comme suit: Présidents: MM. Lamb, Picard, Volterra; Président: M. de la Vallée Poussin; Vice-Président: M. Young; Secrétaires: MM. de Donder, Koenigs, Petrowich, Reina.

Il se divisera en quatre sections qui seront subdivisées elles-mêmes en autant de sous-sections que le nombre et la nature des Communications l'exigeront.

Section I.—Arithmétique—Algèbre—Analyse.

Section II.—Géométrie.

Section III.—Mécanique—Physique mathématique — Mathématiques appliquées.

Section IV.—Questions philosophiques, historiques, pédagogiques.

Des comptes rendus comportant au moins un résumé des travaux du Congrès seront envoyés à chaque souscripteur.

Un programme détaillé donnant les indications concernant le voyage et le logement ainsi que celles relatives aux réceptions et excursions organisées sera publié ultérieurement.

E. PICARD,
Le Président du Comité National Français.

Renseignements Complémentires.

I. Les droits d'inscription en qualité de Membre du Congrès sont fixés à 60 francs par personne payables à M. Valiron, Trésorier du Congrès (52, Allée de la Robertsau, Strasbourg).

Moyennant une cotisation de 30 francs, toute personne de la famille d'un des membres aura droit aux mêmes privilèges que celui-ci, à l'exception de l'envoi d'un exemplaire des comptes rendus.

II. Toute personne désireuse de faire une ou plusieurs Communications au Congrès est priée d'en aviser M. Kœnigs, Secrétaire général du Comité National Français (96, Boulevard Raspail, Paris) et de lui en faire connaitre le sujet avant le 1er Juillet.

III. Pour toute demande de renseignements, s'adresser soit à M. Kœnigs, soit à M. Villat, Président du Comité local d'organisation du Congrès (11, rue due Maréchal-Pétain, à Strasbourg), soit à M. Galbrun, Secrétaire du Comité National Français (14, avenue Émile-Deschanel, Paris).

Millions of Americans are thinking today along wrong lines. Their trend of thought and action is toward extravagance rather than toward production, toward luxuries rather than toward necessities, toward spending rather than saving and toward speculation rather than toward safe productive investment.

It requires no deep knowledge of economics to deduce the danger from such a trend of thought and action, not only to the individual but to the nation and to the world. The inexorable laws of supply and demand still function. Conditions can only return to the safe and the normal when increased production and decreased consumption restore the equilibrium of prosperity; when spending is met by saving; when the desire to get rich quick is tempered by safety and sane profit.

But thought must precede action. It is necessary for America to think right in order that her citizens may act right. To guide the trend of public thought is both the duty and the privilege of the university men of America. They must teach the lesson of thrift and economy, of working and saving; lay the foundations of sound economic knowledge and practise. There is but one other way for America to learn sound financial habits, that is by experience through economic and financial crash which will bring untold suffering in its trail.